Chapter 31A - Electromagnetic Induction

A PowerPoint Presentation by

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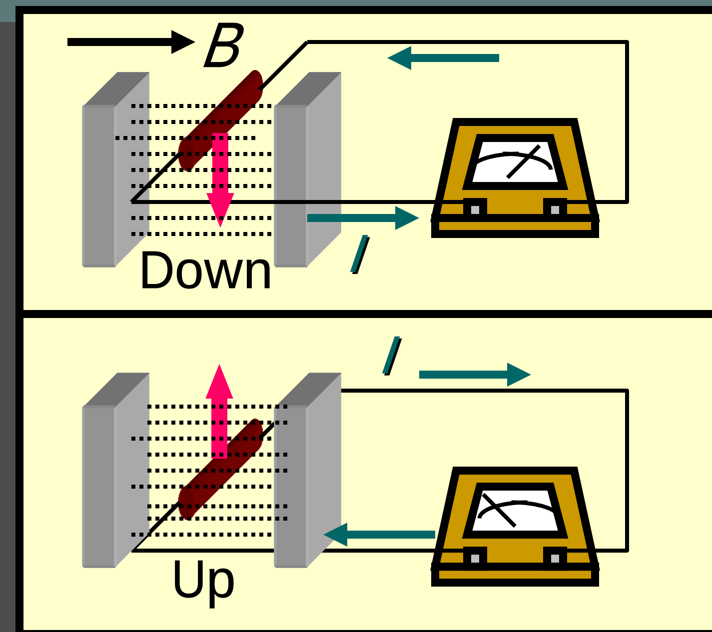
Objectives: After completing this module, you should be able to:

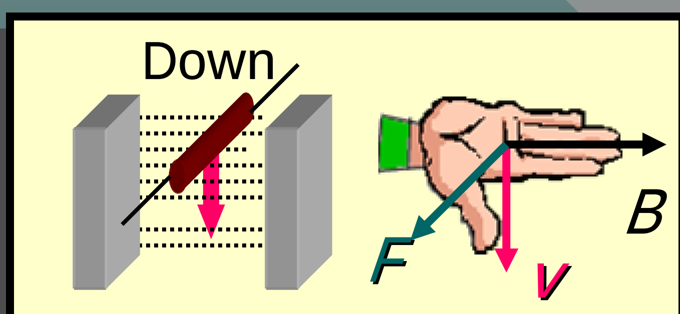
- Calculate the magnitude and direction of the induced current or emf in a conductor moving with respect to a given B-field.
- Calculate the magnetic flux through an area in a given B-field.
- Apply Lenz's law and the right-hand rule to determine directions of induced emf.
- Describe the operation and use of ac and dc generators or motors.

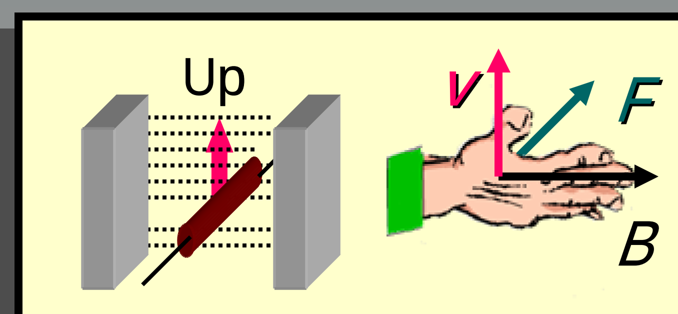
Induced Current

When a conductor moves across flux lines, magnetic forces on the free electrons induce an electric current.

Right-hand force rule shows current outward for down and inward for up motion. (Verify)



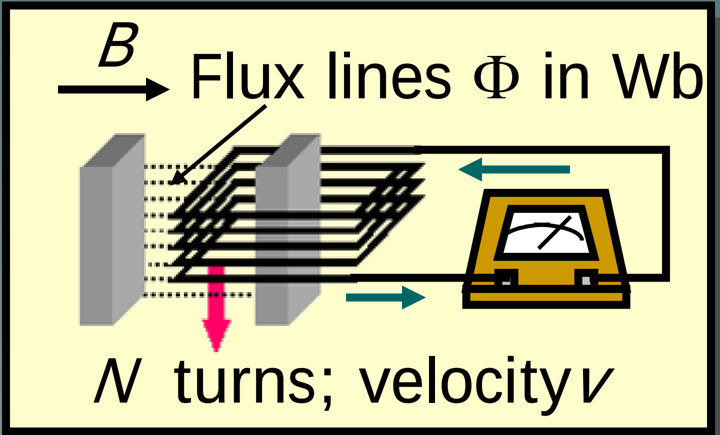




Induced EMF: Observations

<u>Faraday's observations:</u>

- Relative motion induces emf.
- Direction of emf depends on direction of motion.
- Emf is proportional to rate at which lines are cut (*v*).
- Emf is proportional to the number of turns *N*.



Faraday's Law:

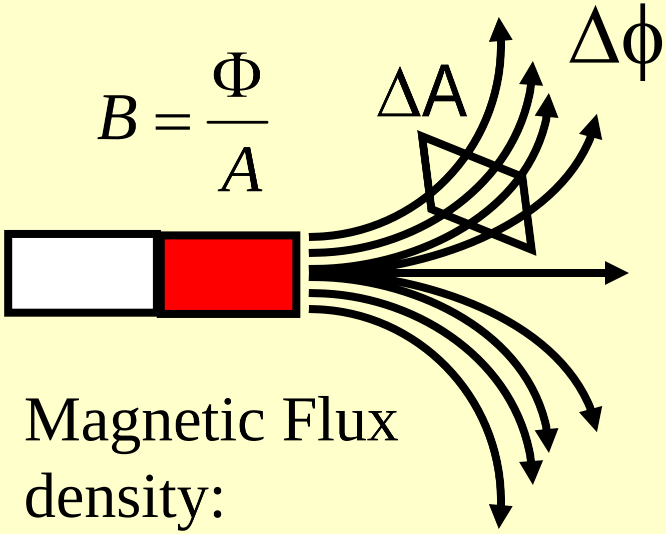
$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$$

The negative sign means that $\mathcal{E}$ opposes its cause.

Magnetic Flux Density

- Magnetic flux lines Φ are continuous and closed.
- Direction is that of the B vector at any point.

$$B = \frac{\Phi}{A}$$



Magnetic Flux density:

When area A is perpendicular to flux:

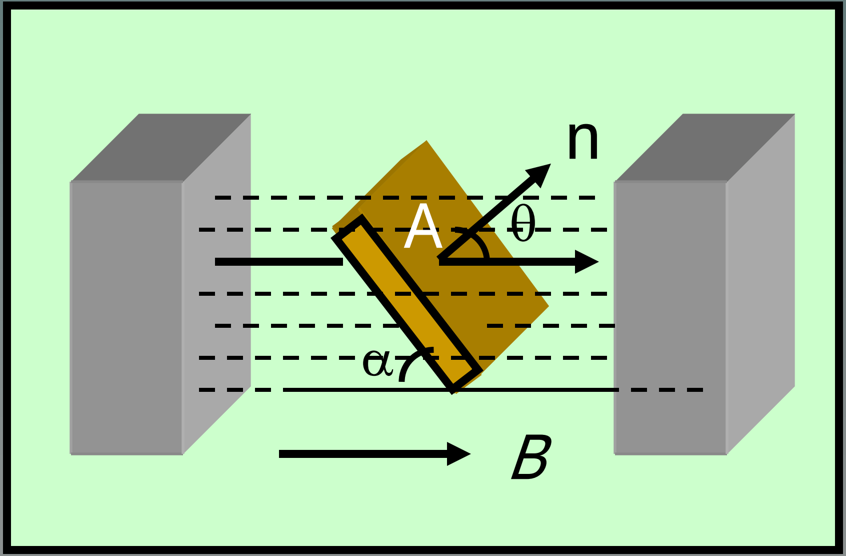
$$B = \frac{\Phi}{A}; \quad \Phi = BA$$

The unit of flux density is the weber per square meter.

Calculating Flux When Area is Not Perpendicular to Field

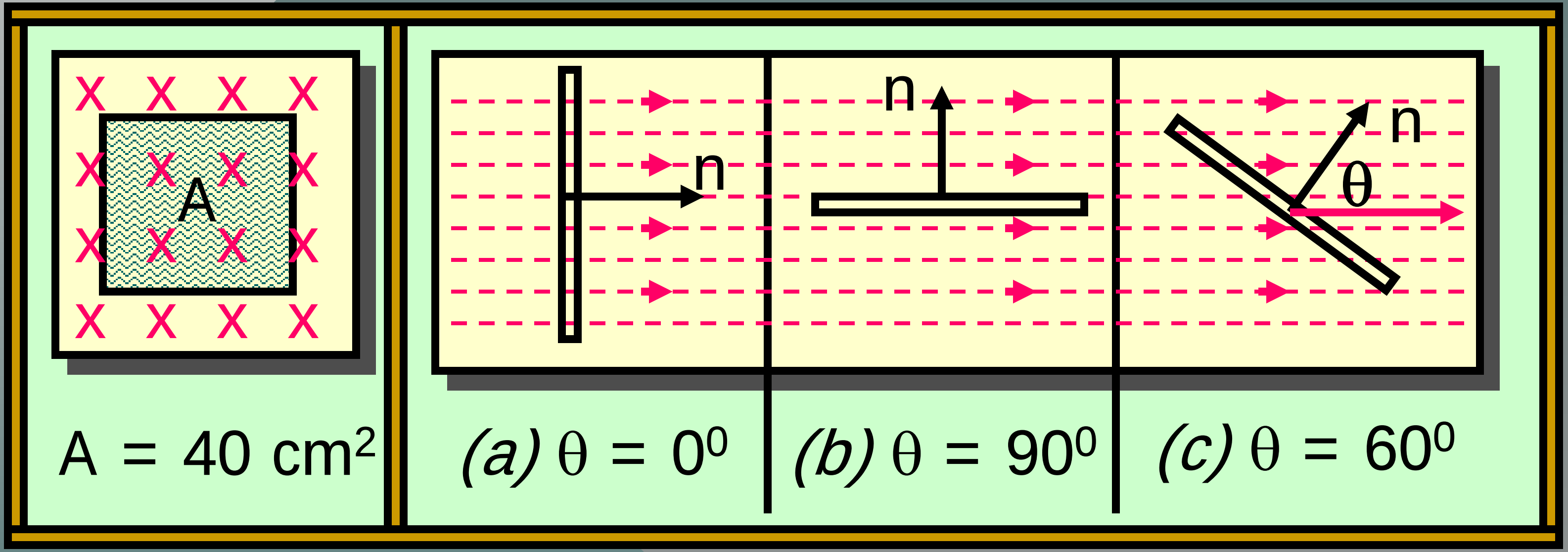
The flux penetrating the area A when the normal vector n makes an angle of θ with the B-field is:

$$\Phi = BA\cos\theta$$



The angle θ is the complement of the angle α that the plane of the area makes with B field. (Cos θ = Sin α)

Example 1: A current loop has an area of 40 cm^2 and is placed in a 3-T B-field at the given angles. Find the flux Φ through the loop in each case.



(a) $\Phi = BA \cos 0^0 = (3\ \text{T})(0.004\ \text{m}^2)(1)$; $\Phi = 12.0$ mWb

(b) $\Phi = BA \cos 90^0 = (3\ \text{T})(0.004\ \text{m}^2)(0)$; $\Phi = 0$ mWb

(c) $\Phi = BA \cos 60^0 = (3\ \text{T})(0.004\ \text{m}^2)(0.5)$; $\Phi = 6.00$ mWb

Application of Faraday's Law

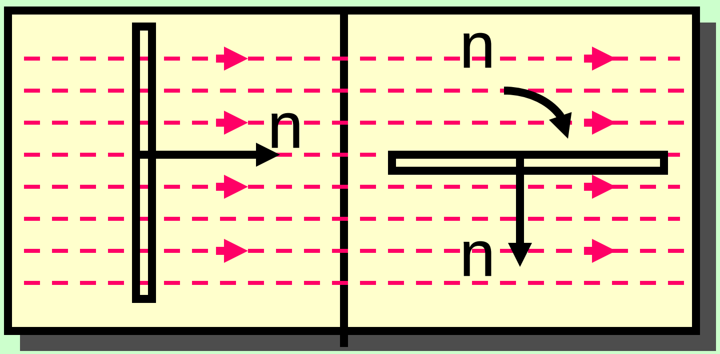
Faraday's Law:

$$\mathcal{E} = -N \frac{\Delta\Phi}{\Delta t}$$

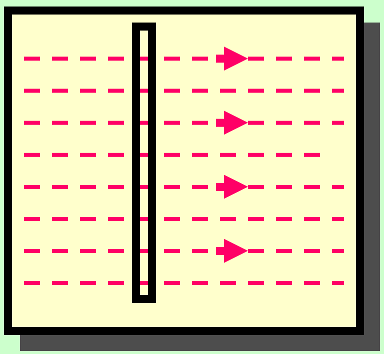
A change in flux $\Delta\Phi$ can occur by a change in area or by a change in the B-field:

$\Delta\Phi = B\,\Delta A$ $\quad\quad$ $\Delta\Phi = A\,\Delta B$

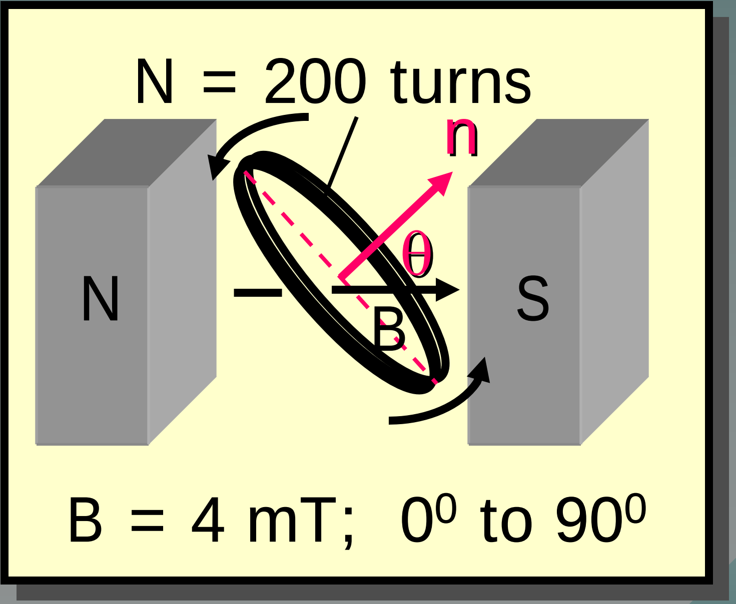
Rotating loop = B ΔA



Loop at rest = A ΔB



Example 2: A coil has 200 turns of area 30 cm^2. It flips from vertical to horizontal position in a time of 0.03 s. What is the induced emf if the constant B-field is 4 mT?



ΔA = 30 cm^2 – 0 = 30 cm^2

$\Delta\Phi = B\,\Delta A$ = (3 mT)(30 cm^2)

$\Delta\Phi$ = (0.004 T)(0.0030 m^2)

$\Delta\Phi = 1.2 \times 10^{-5}$ Wb

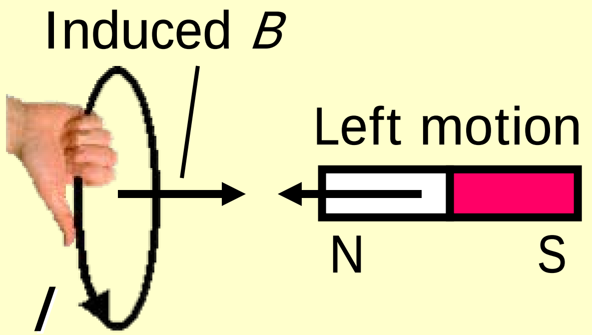
$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -(200)\frac{1.2 \text{ x } 10^{-5}\text{Wb}}{0.03 \text{ s}}$$

$\mathcal{E}$ = -0.080 V

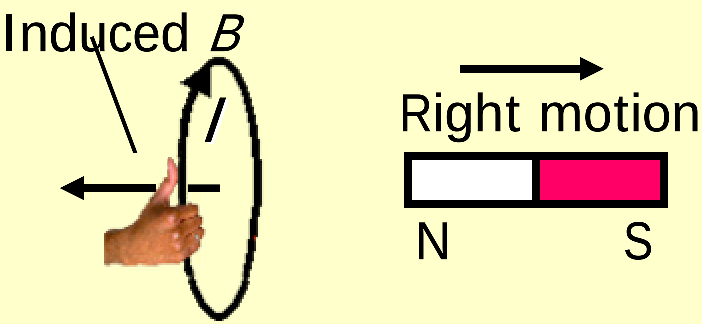
The negative sign indicates the polarity of the voltage.

Lenz's Law

Lenz's law: An induced current will be in such a direction as to produce a magnetic field that will oppose the motion of the magnetic field that is producing it.



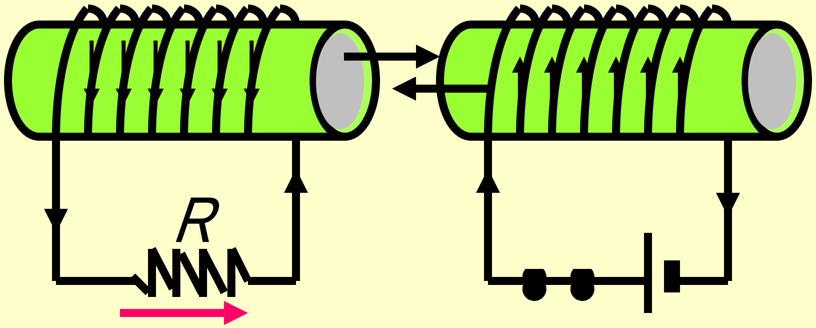
Flux increasing to left induces loop flux to the right.



Flux decreasing by right move induces loop flux to the left.

Example 3: Use Lenz's law to determine direction of induced current through *R* if switch is closed for circuit below (*B* increasing).

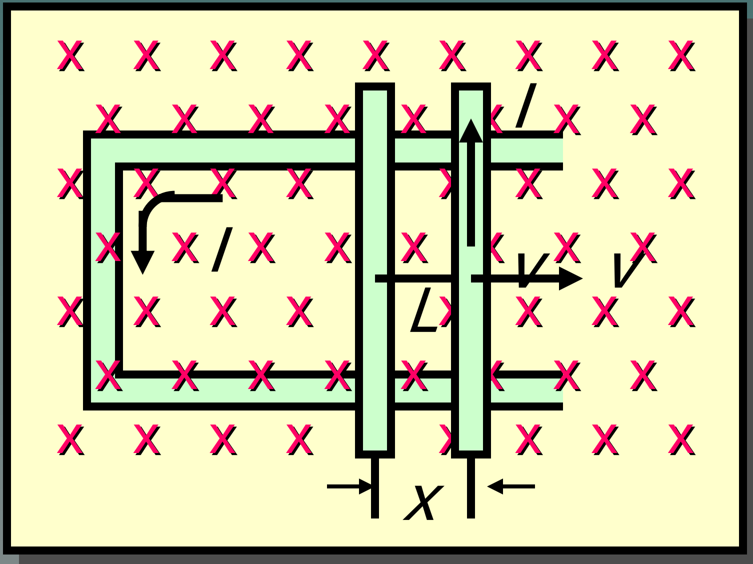
Close switch. Then what is direction of induced current?



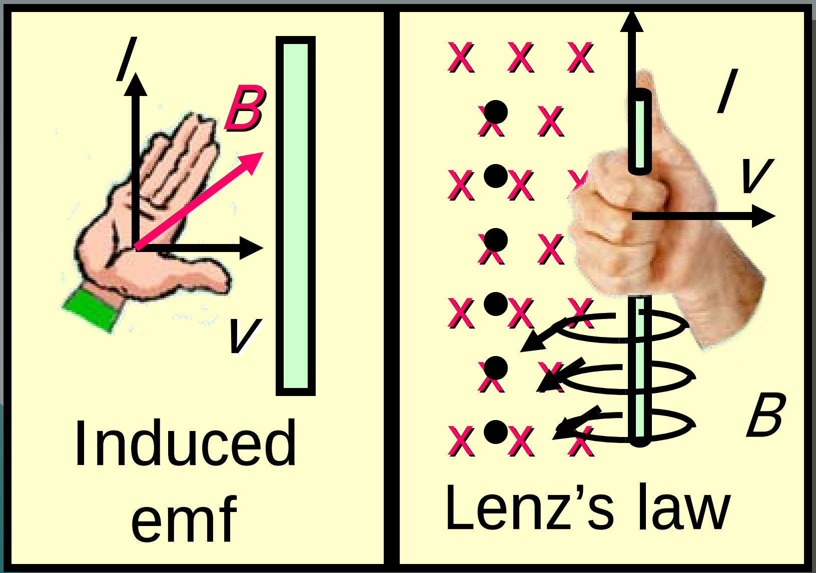
The rising current in right circuit causes flux to increase to the left, inducing current in left circuit that must produce a rightward field to oppose motion. Hence current *I* through resistor *R* is to the right as shown.

Directions of Forces and EMFs

An emf $\mathcal{E}$ is induced by moving wire at velocity *v* in constant *B* field. Note direction of *I*.



From Lenz's law, we see that a reverse field (out) is created. This field causes a leftward force on the wire that offers resistance to the motion. Use right-hand force rule to show this.



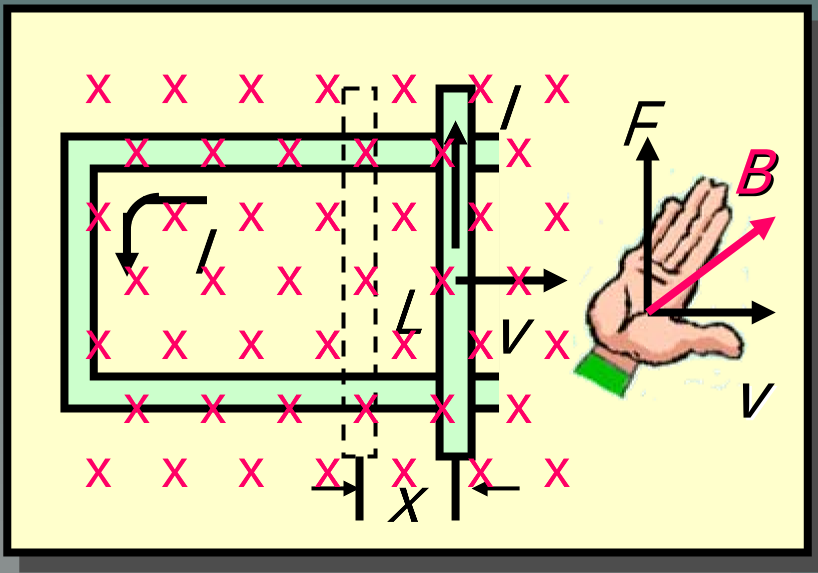
Motional EMF in a Wire

Force *F* on charge *q* in wire:

F = qvB; Work = FL = qvBL

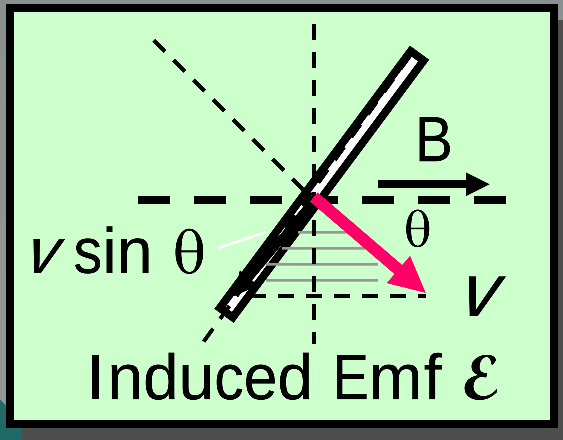
$$\mathcal{E} = \frac{Work}{q} = \frac{\not{q}vBL}{\not{q}}$$

EMF: $\mathcal{E} = BLv$



If wire of length L moves with velocity *v* an angle θ with *B:*

$$\mathcal{E} = BLv\sin\theta$$

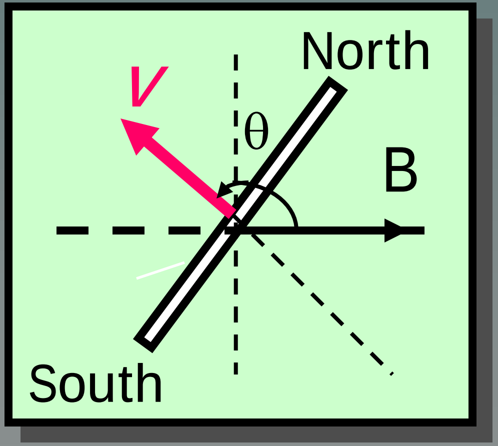


Example 4: A 0.20-m length of wire moves at a constant speed of 5 m/s in at 140^0 with a 0.4-T *B*-Field. What is the magnitude and direction of the induced emf in the wire?

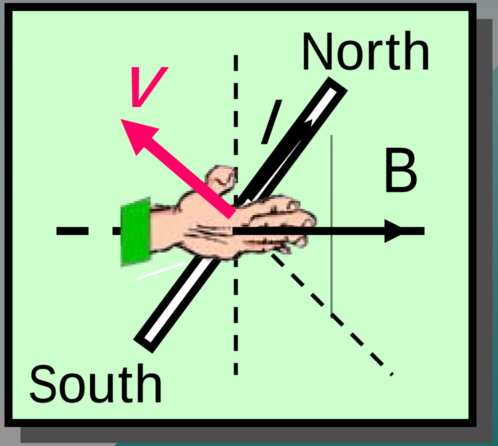
$$\mathcal{E} = BLv\sin\theta$$

$$\mathcal{E} = (0.4\text{ T})(0.20\text{ m})(5\text{ m/s})\sin 140^0$$

$$\mathcal{E} = -0.257\text{ V}$$

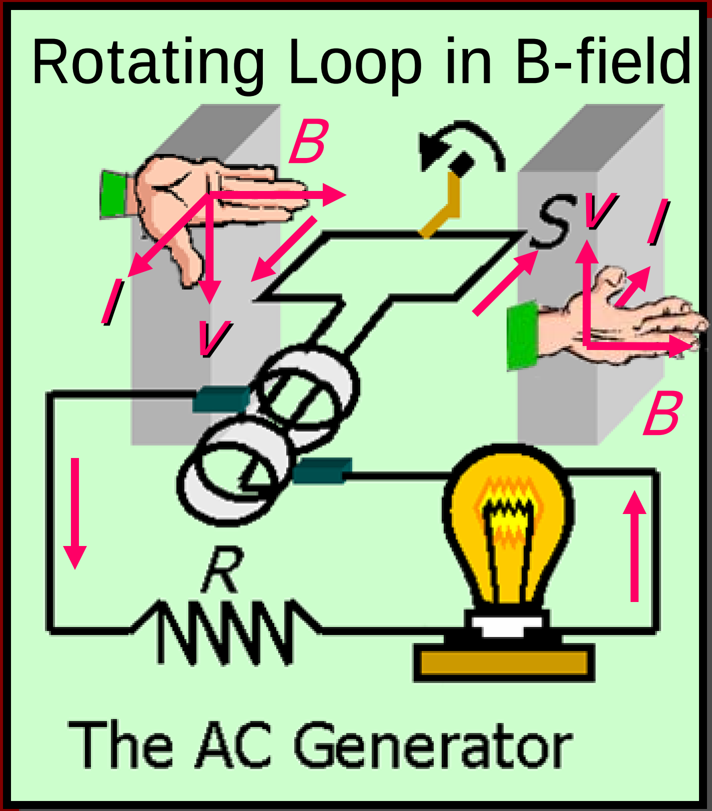


Using right-hand rule, point fingers to right, thumb along velocity, and hand pushes in direction of induced emf—to the north in the diagram.



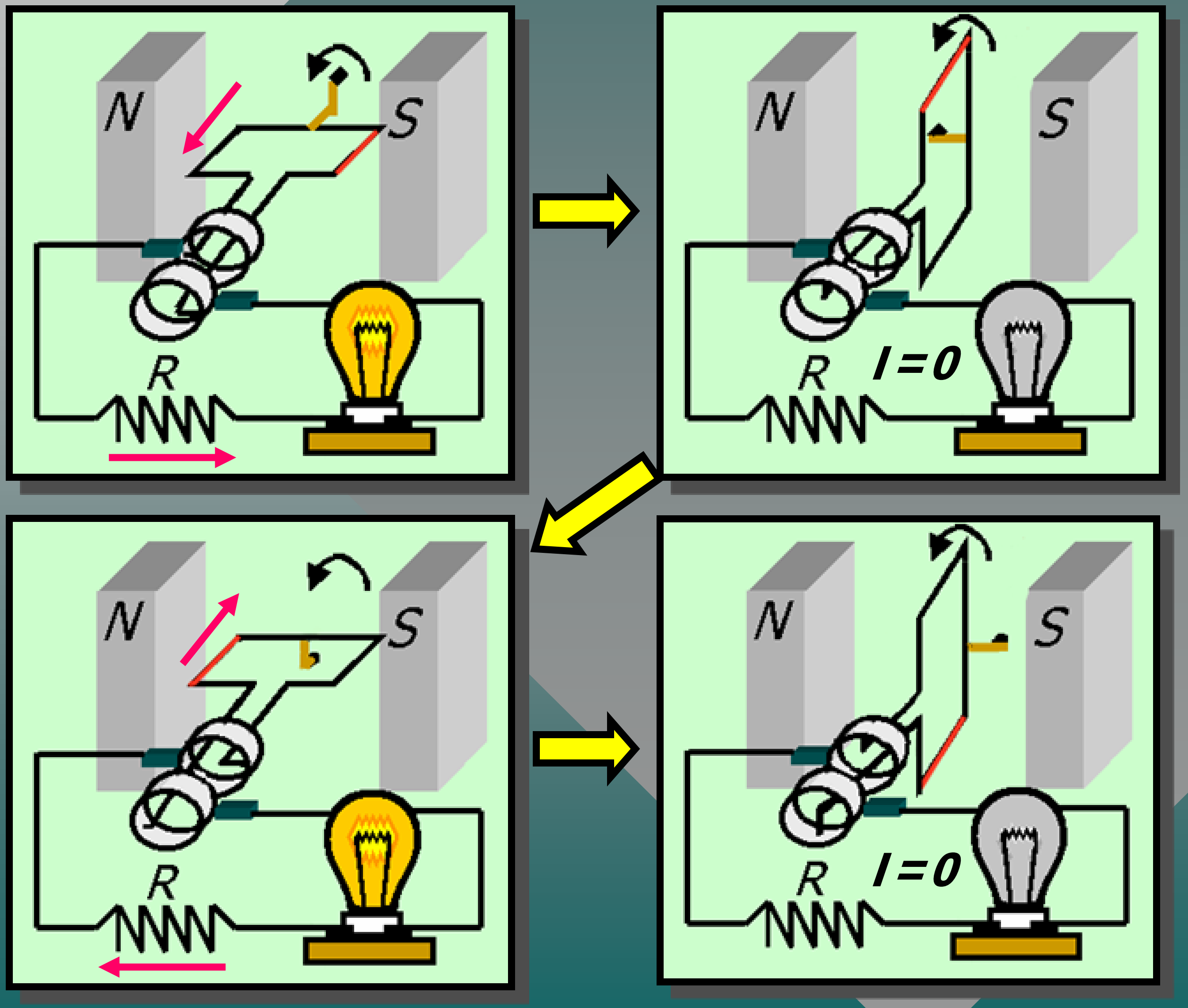
The AC Generator

- An alternating AC current is produced by rotating a loop in a constant *B*-field.
- Current on left is outward by right-hand rule.
- The right segment has an inward current.
- When loop is vertical, the current is zero.



I in *R* is right, zero, left, and then zero as loop rotates.

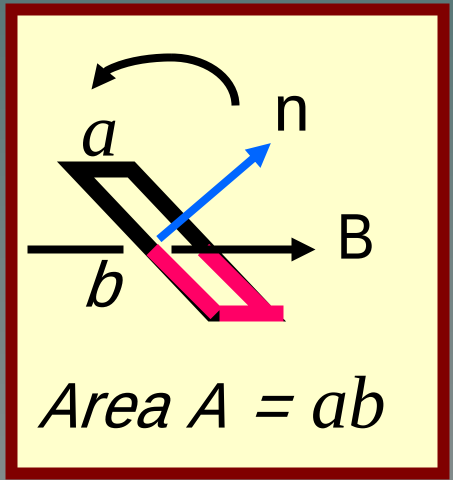
Operation of AC Generator

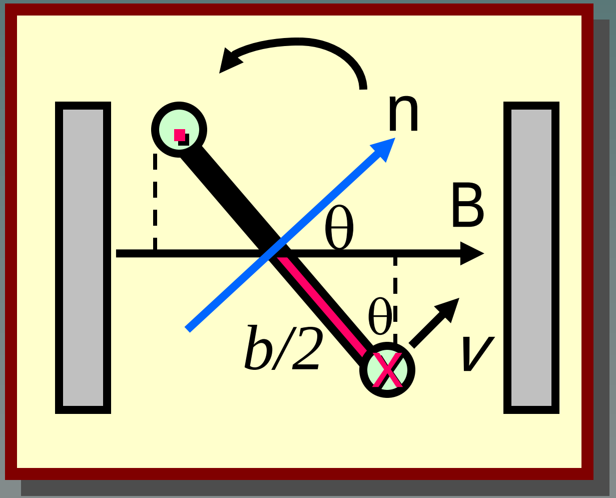


Calculating Induced EMF

Rectangular loop $a \times b$

Each segment a has constant velocity v.



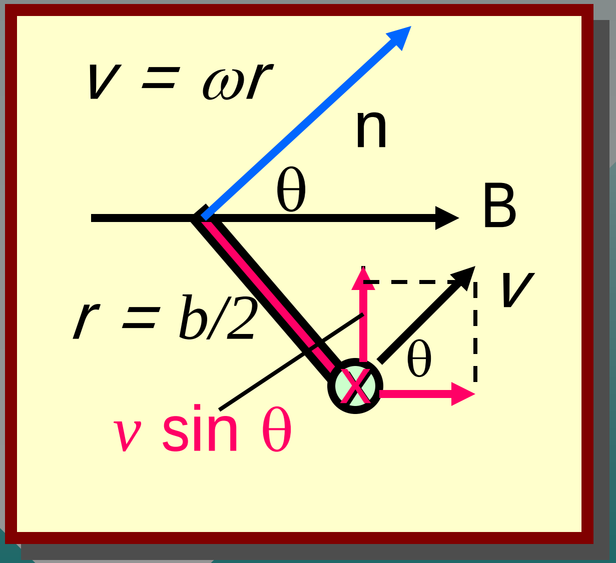


Both segments a moving with v at angle θ with B gives emf:

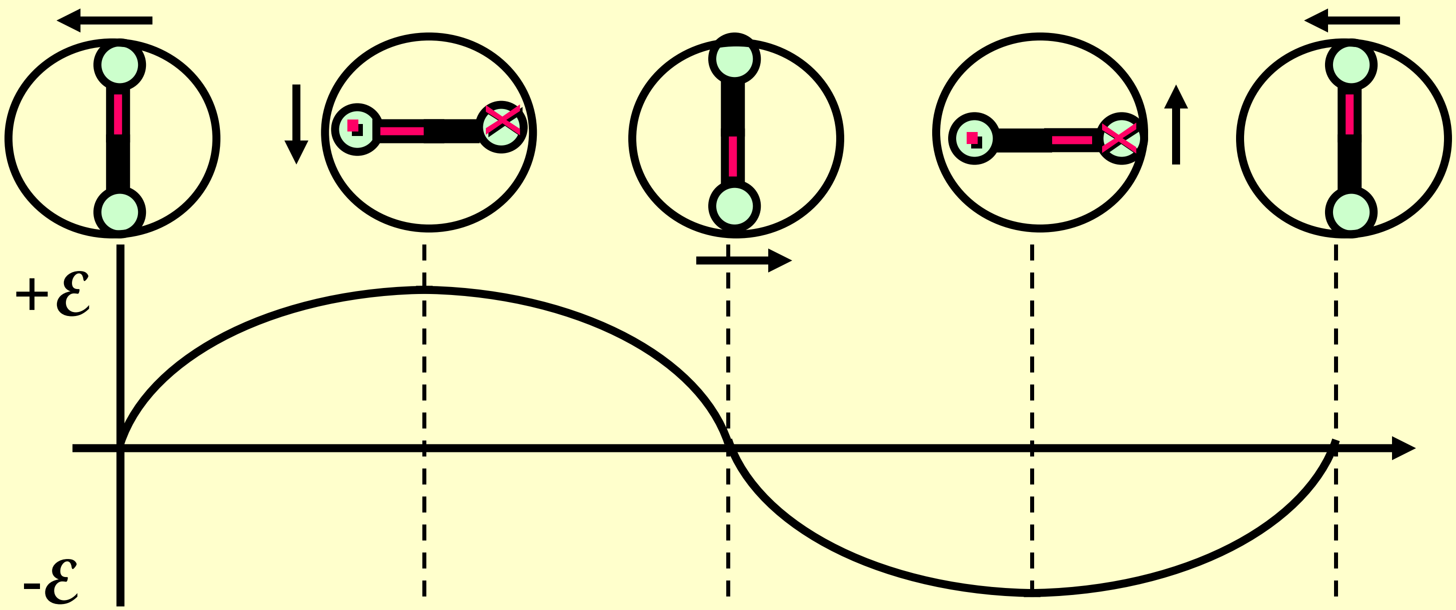
$$\mathcal{E} = Bav\sin\theta; \quad v = \omega r = \omega\left(b/2\right)$$

$$\mathcal{E}_T = 2Ba\left(\omega\, b/2\right)\sin\theta$$

$$\boxed{\mathcal{E}_T = BA\omega\sin\theta}$$



Sinusoidal Current of Generator



The emf varies sinusoidally with max and min emf

For *N* turns, the EMF is: $\mathcal{E} = NBA\omega \sin\theta$

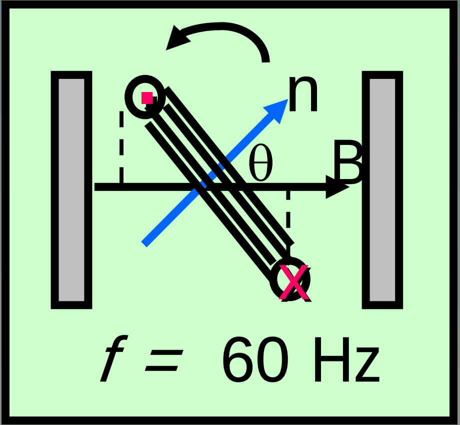
Example 5: An ac generator has 12 turns of wire of area 0.08 m². The loop rotates in a magnetic field of 0.3 T at a frequency of 60 Hz. Find the maximum induced emf.

$\omega = 2\pi f = 2\pi(60 \text{ Hz}) = 377 \text{ rad/s}$

Emf is maximum when $\theta = 90^0$.

$$\mathcal{E}_{max} = NBA\omega; \quad \text{Since} \quad \sin\theta = 1$$

$$\mathcal{E}_{max} = (12)(0.3 \text{ T})(.08 \text{ m}^2)(377 \text{ rad/s})$$

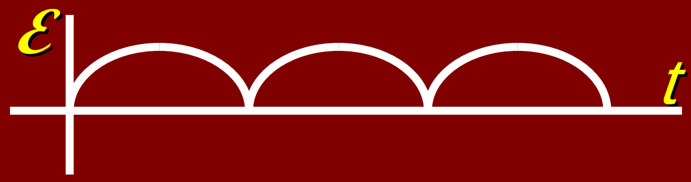


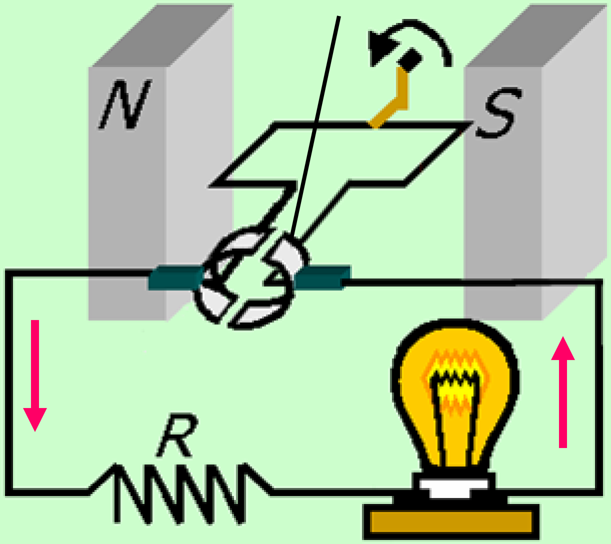
The maximum emf generated is therefore: $\mathcal{E}_{max} = 109 \text{ V}$

If the resistance is known, then Ohm's law ($V = IR$) can be applied to find the maximum induced current.

The DC Generator

The simple ac generator can be converted to a dc generator by using a single split-ring commutator to reverse connections twice per revolution.





For the dc generator: The emf fluctuates in magnitude, but always has the same direction (polarity).

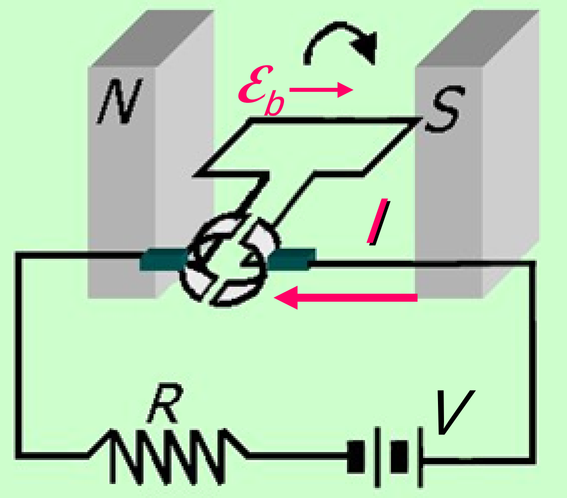
The Electric Motor

In a simple electric motor, a current loop experiences a torque which produces rotational motion. Such motion induces a back emf to oppose the motion.

Applied voltage – back emf
= net voltage

$$V - \mathcal{E}_b = IR$$

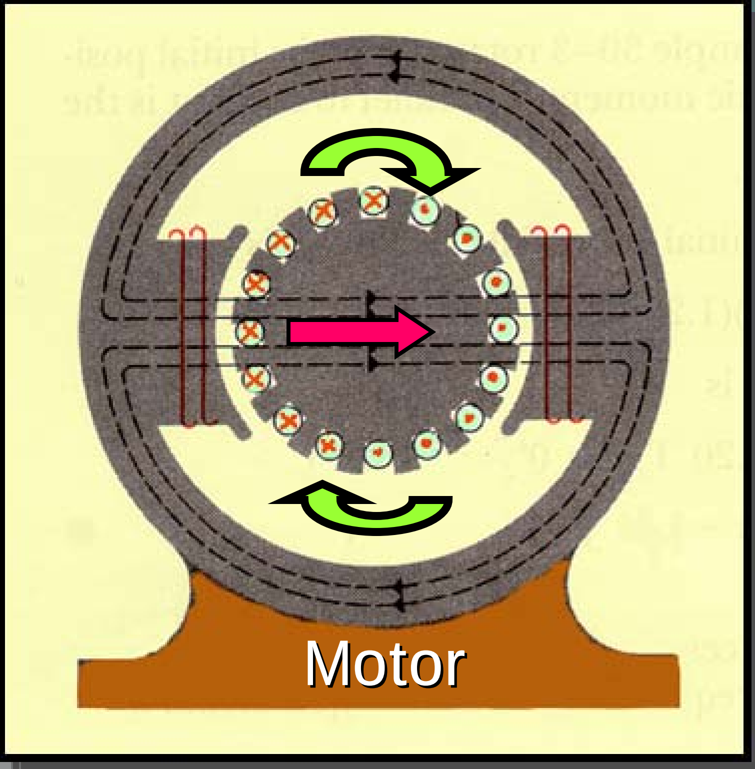
Since back emf $\mathcal{E}_b$ increases with rotational frequency, the starting current is high and the operating current is low: $\mathcal{E}_b = NBA\omega \sin \theta$



Electric Motor

Armature and Field Windings

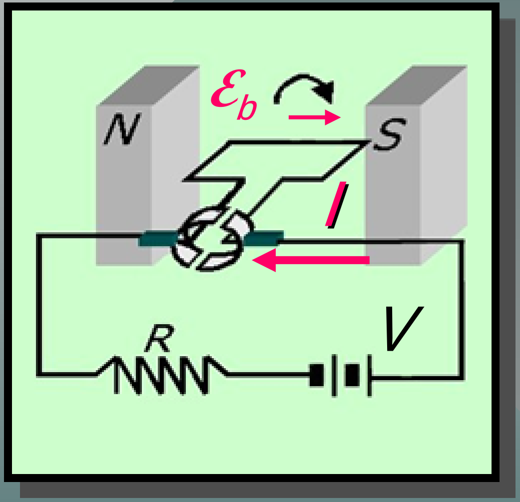
In the <u>commercial motor</u>, many coils of wire around the armature will produce a smooth torque. (Note directions of / in wires.)



<u>Series-Wound Motor:</u> The field and armature wiring are connected in series.

<u>Shunt-Wound Motor:</u> The field windings and the armature windings are connected in parallel.

Example 6: A series-wound dc motor has an internal resistance of 3 Ω. The 120-V supply line draws 4 A when at full speed. What is the emf in the motor and the starting current?



Recall that: $V - \mathcal{E}_b = IR$

$$120 \text{ V} - \mathcal{E}_b = (4 \text{ A})(3\ \Omega)$$

The back emf in motor: $\mathcal{E}_b = 108 \text{ V}$

The starting current I_s is found by noting that $\mathcal{E}_b = 0$ in beginning (armature has not started rotating).

$$120 \text{ V} - 0 = I_s(3\ \Omega)$$

$I_s = 40 \text{ A}$

Summary

Faraday's Law:

$$\mathcal{E} = -N \frac{\Delta\Phi}{\Delta t}$$

A change in flux $\Delta\Phi$ can occur by a change in area or by a change in the B-field:

$\Delta\Phi = B\,\Delta A$ $\Delta\Phi = A\,\Delta B$

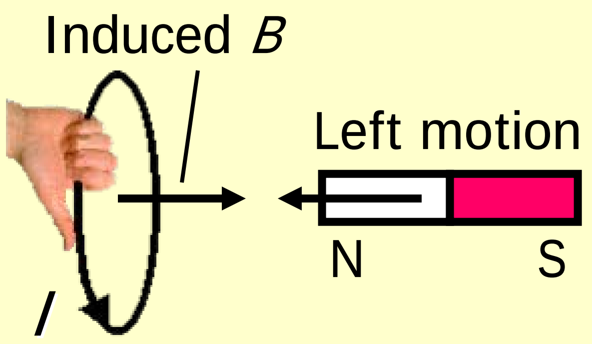
Calculating flux through an area in a B-field:

$$B = \frac{\Phi}{A}; \quad \Phi = BA$$

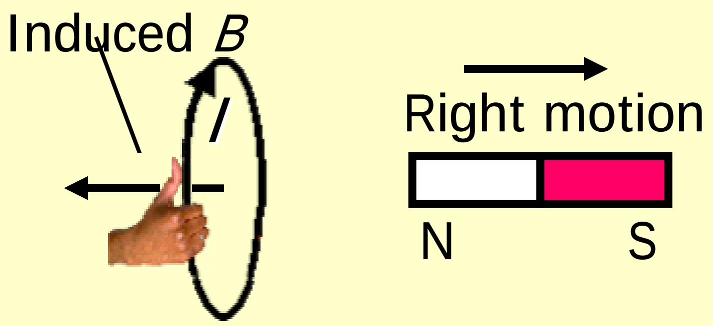
$$\Phi = BA\cos\theta$$

Summary (Cont.)

Lenz's law: An induced current will be in such a direction as to produce a magnetic field that will oppose the motion of the magnetic field that is producing it.



Flux increasing to left induces loop flux to the right.

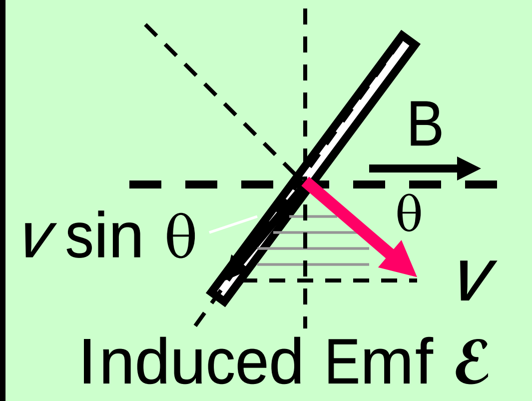


Flux decreasing by right move induces loop flux to the left.

Summary (Cont.)

An emf is induced by a wire moving with a velocity *v* at an angle θ with a B-field.

$$\mathcal{E} = BLv\sin\theta$$

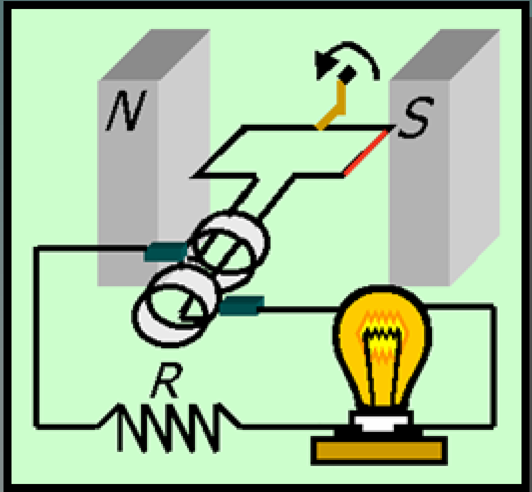


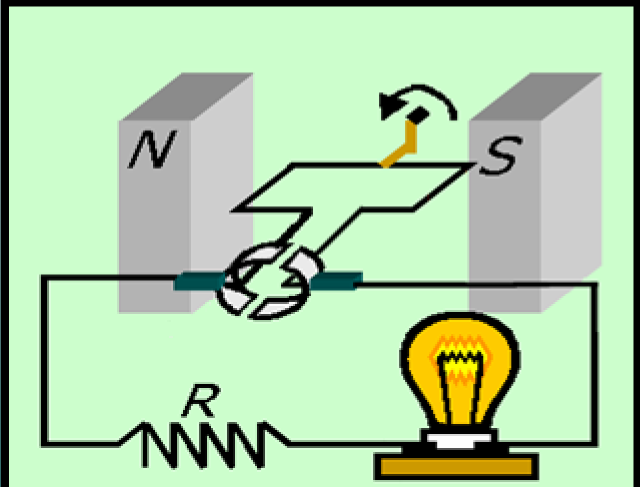
In general for a coil of N turns of area A rotating with a frequency in a B-field, the generated emf is given by the following relationship:

For *N* turns, the EMF is: $\mathcal{E} = NBA\omega\sin\theta$

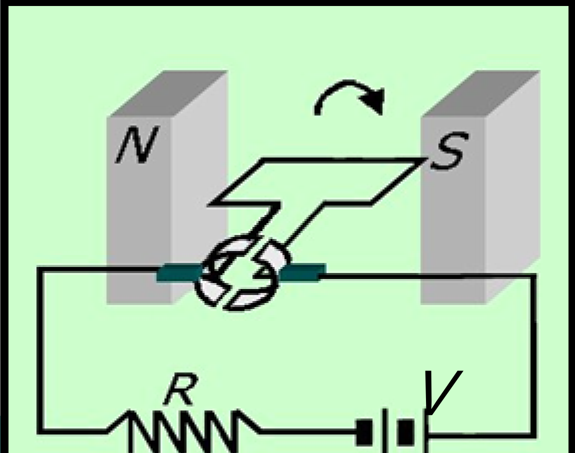
Summary (Cont.)

The ac generator is shown to the right. The dc generator and a dc motor are shown below:





DC Generator



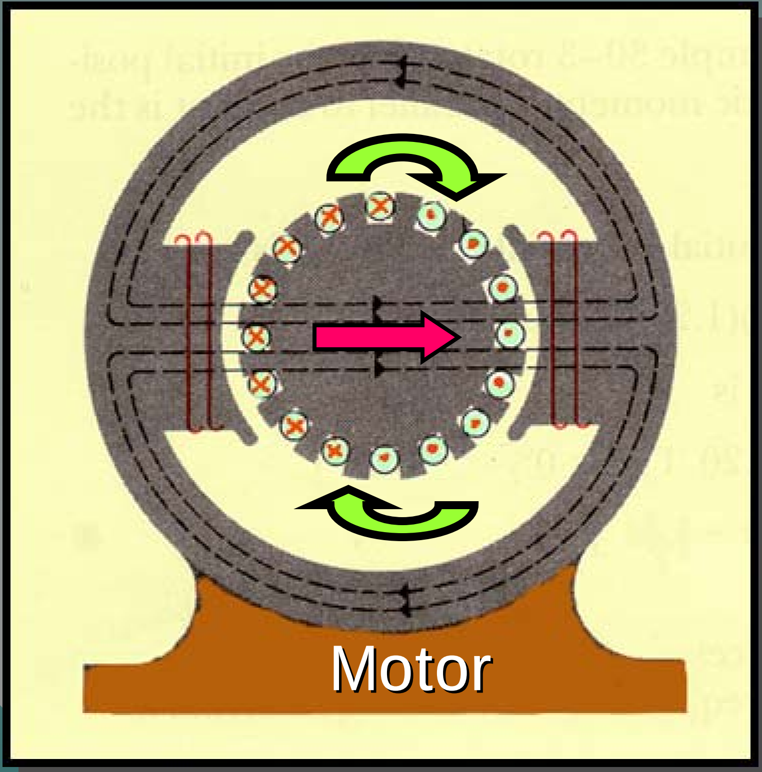
Electric Motor

Summary (Cont.)

The rotor generates a back emf in the operation of a motor that reduces the applied voltage. The following relationship exists:

Applied voltage – back emf = net voltage

$$V - \mathcal{E}_b = IR$$



CONCLUSION: Chapter 31A Electromagnetic Induction